Home Market Effects and Increasing Returns with Non-Constant Marginal Costs

Nahyeon Bak*
Amazon.com

Daisoon Kim†
North Carolina State University

Mishita Mehra‡
University of Richmond

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Abstract

We reexamine the role of increasing returns in production, central to trade and economic geography theories, focusing on the home market effect. We extend the conventional multi-industry new trade model to introduce (i) non-constant marginal costs and (ii) non-homothetic production in factors. If factors that are more (less) intensively used in fixed costs than variable costs are also those with higher relative prices in large countries compared to small countries, then large countries exhibit larger (smaller) firm sizes and specialize in industries with decreasing (increasing) marginal costs. Notably, different levels of fixed costs have limited impact on these patterns.

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Keywords: Home market effect, Economies of scale, Non-constant marginal costs, Non-homothetic production, Factor endowment.

*Core AI, Amazon.com, 345 Boren Avenue, Seattle, WA 98144, US. Email: nahyeonbak@gmail.com

†Corresponding author. Department of Economics, Poole College of Management, North Carolina State University, Nelson Hall 4114, Raleigh, NC 27695, US. Email: dkim29@ncsu.edu

‡Department of Economics, Robins School of Business, University of Richmond, RSB 353, Richmond, VA 23173, US. Email: mishita.mehra@richmond.edu

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1. Introduction

This paper evaluates the theoretical robustness of the Home Market Effect (HME), whereby countries with higher market access specialize in industries characterized by scale economies in a trade equilibrium with trade costs and market segmentation. The existing literature mostly relies on a constant marginal cost and a fixed cost, which represents the simplest way to generate increasing returns to scale within the representative firm. Our study breaks away from this tradition in two distinct ways.

First, the main novelty of the model is the possibility of variable returns to scale in production. We allow marginal costs to be an isoelastic function of output, while maintaining the assumption of fixed costs. A single parameter α (the output elasticity of variable costs) succinctly captures the essence of the argument. This extension encompasses the standard case of a constant marginal cost ($\alpha = 1$). Within this framework, firms encounter increasing returns to scale across all levels of output when their marginal cost function is non-increasing in production ($\alpha \geq 1$). However, if marginal costs increase with output ($\alpha < 1$), firms produce under variable returns to scale, contingent on the output level: increasing returns at low output levels, but decreasing returns at high output levels.

Secondly, we allow for non-homothetic production in labor and capital, the two factors of production. Fixed (non-production or entry) costs and variable (production) costs are Cobb-Douglas combinations of labor and capital, but the Cobb-Douglas shares can vary across factors. The production functions in our paper encompass both homothetic (Helpman and Krugman 1985) and non-homothetic (Flam and Helpman 1987) models as special cases. Additionally, we differentiate the two factors by their relative endowments. The capital-to-labor ratio is higher in large countries compared to small countries, i.e. labor is scarce relative to capital in large countries. Both factors are immobile across countries.

We incorporate these extensions into a two-country, multi-industry framework similar to the model developed by Hanson and Xiang (2004). The economy is made of two countries, segmented by trade costs, and a continuum of monopolistically competitive industries. Each industry is made of identical firms. Industries differ according to technology (the level of fixed costs, the parameter governing the slope of the marginal cost, and the input shares in variable and fixed costs), preferences (the elasticity of substitution between any pair of varieties), and trade costs. By introducing a more general production structure with non-constant marginal costs and non-homothetic production, our model allows for a deeper exploration of how economies of scale interact with the HME.

Our model yields a key prediction: When the fixed cost structure of an industry relies more heavily on factors that are relatively more expensive in large countries compared to variable costs, these larger countries will tend to have larger firms and specialize in industries with decreasing marginal costs. Conversely, if the fixed cost structure is intensive in factors that are relatively cheaper in large countries, the opposite pattern emerges. This creates a general equilibrium feedback effect on firm scale, where firms are larger in the large country for some industries but not necessarily for others. In contrast, under homothetic production functions (where $\tilde{\theta} = \theta$), firm production scales remain identical between countries within each industry, regardless of differences in factor prices and market size.

Due to trade costs, larger countries become more attractive locations for firms, leading to higher factor prices compared to smaller countries. Without loss of generality, suppose wages increase more than capital rental rates in larger countries, resulting in a higher labor-to-capital ratio. In our model, this occurs due to different factor endowments. Large countries have a higher capital-to-labor ratio compared to small countries, i.e., labor and capital are relatively scarce and abundant, respectively, in larger countries.[1]

Now, let's explore how these relative factor prices interact with industry characteristics to influence the HME. Industries with decreasing marginal costs ($\alpha > 1$) and a higher share of labor costs in fixed costs compared to variable costs ($\tilde{\theta} > \theta$) create a scenario where firms in large countries benefit from economies of scale. This advantage arises because firms only experience these cost savings if they can produce at a larger scale. For industries with fixed production costs with a higher labor input share compared to the variable production cost, the higher labor-to-capital price ratio translates into higher entry/operation costs relative to production costs for large countries. As a result, these industries are less incentivized to expand through the extensive margin (by increasing the number of firms) relative to the intensive margin (by increasing production within each firm). This encourages fewer firms to enter the market, leading to a larger production scale for each firm, which translates into gains from economics of scale in larger countries.

Conversely, internal economics of scale from decreasing marginal costs translate into disadvantages for firms in large countries when the labor is more expensive (scarce) in larger countries, and

[1] This disparity in factor endowments leads to a significant difference in capital-labor ratios between the two economies, with the Home country exhibiting a substantially higher ratio.

the labor input share is smaller in the fixed cost function relative to the variable cost function. In this case, higher firm entry implies that individual firm sizes would remain relatively small in the larger country, despite the larger market. Then, large countries would not tend to specialize in such industries, despite the presence of internal economics of scale.

In summary, the interplay between the output elasticity (α) and the relative input intensities of fixed and variable costs ($\tilde{\theta} - \theta$) is crucial for how non-constant marginal costs influence the HME. On the other hand, fixed costs themselves play a limited role in shaping HME patterns across industries, despite being an important ingredient in the model with monopolistic competition and isoelastic demand. This understanding of non-constant marginal costs, its relationship with input intensities, and the role of fixed costs contributes to the HME literature.

Our paper contributes to a wide literature on HME that is one of the main predictions of the new trade theory. Conventional literature such as Krugman (1980); Helpman and Krugman (1985); Davis (1998) focuses on very broadly defined industry-level analysis. Large countries tend to specialize in the manufacturing sector rather than an outside sector (for example, agricultural sector) when the manufacturing sector faces economics of scale, differentiated products, and trade costs. Related empirical studies by Feenstra, Markusen and Rose (1998); Davis and Weinstein (1999); Head and Ries (2001) test the HME based on broadly defined industries. In contrast, Costinot et al. (2019) study HME using detailed data within the pharmaceutical industry.

Using multi-industry trade frameworks, several studies, such as Hanson and Xiang (2004); Pham, Lovely and Mitra (2014); Chen and Zeng (2017), try to understand and identify the marginal impacts of demand-side features of industries on the HME. Hanson and Xiang (2004) find evidence of a positive impact of transport costs and product differentiation on industry specialization in larger countries. Recently, Zhelobodko et al. (2012) attempt to highlight a new angle on HME by employing non-constant elasticity of substitution (non-CES) preferences.

Our paper also complements several international trade studies on increasing returns in production, such as Grossman and Rossi-Hansberg (2010); Lyn and Rodriguez-Clare (2013), who study implications of national/sectoral level external economics of scale in trade and welfare. Closely related to our work, Panagariya (1981); Holmes and Stevens (2005) theoretically study the role of economics of scale on trade patterns. Panagariya (1981)'s comparative advantage model predicts that a small country is specialized in industries with small economics of scale. In Holmes and Stevens (2005)'s model, goods with small economics of scale are not traded. Large countries become a net-exporter in industries with large economics of scale. However, their mechanisms differ from the original spirit of HME, as they do not consider firm entry.

The recent literature on international economics has paid attention to non-constant marginal costs. Even though constant marginal costs with fixed costs are widely used in international economics, it has some drawbacks, especially in multi-industry models. First, fixed costs cannot generate dis-economics of scale, but some industries' average costs increase with quantity. Second, Kim (2021) documents that variations in economics of scale across the US manufacturing industries are mainly associated with marginal costs rather than fixed costs. Third, an individual firm's decisions in domestic and export markets are not separable, which is evidence of non-constant marginal costs (e.g., Vannoorenberghe 2012; Soderbery 2014; Berman, Berthou and Hericourt 2015; Almunia et al. 2018; Kim 2021).

Finally, increasing returns also play an important role in geographic specialization of an industry/firm, e.g., Hanson (2001); Ottaviano and Thisse (2001); Potter and Watts (2010), and many others including HME literature on geographical economics mentioned below. Recently, Bucci and Ushchev (2020); Bond-Smith (2021) focus on increasing returns and their implications on economic geography. In economic geography theories such as Hanson (2001); Holmes and Stevens (2004); Robert-Nicoud (2005) among many others, HME also plays a key mechanism in shaping geographic specialization. Baldwin and Okubo (2005) and Wiberg (2009) show that firm productivity heterogeneity and lobbying weaken the HME channel, respectively. In most economic geography theories mentioned above, scale economies arise from product differentiation and fixed costs but not from non-constant marginal costs. Holmes and Stevens (2004) explain why large cities specialize in services rather than manufacturing by building an economic geography model with increasing marginal costs besides fixed costs.

The remainder of the paper is structured as follows: Section 2 introduces the model's environment and equilibrium. In Section 3, we highlight the significance of input intensity of variable and fixed costs in examining the impact of non-constant marginal costs. Section 4 outlines the key implication of the model, illustrating how the cost structure shapes industrial HME. Section 5 offers concluding remarks.

2. The Model

This section builds and solves a two-country multi-industry trade model, building upon the framework introduced by Hanson and Xiang (2004). Their work extends the new trade model of Krugman (1979, 1980) by incorporating infinitely many industries with diverse demand-side characteristics, including elasticity of substitution across goods, trade costs, and expenditure shares. A key departure from their model lies in the integration of cost-side heterogeneity across industries.

In our model, all industries are CES-monopolistically competitive, trade costs take Samuelson's iceberg form, and all industries consist of homogeneous firms. Firms in each industry produce under variable returns to scale from fixed and non-constant marginal costs. Firms' production functions are potentially non-homothetic in the two factors of production, labor and capital. Both factors are immobile across borders.

Each industry is characterized by the following parameters: α (elasticity of variable costs with respect to output, known as output elasticity, which governs the scope for non-constant marginal costs), σ (elasticity of substitution among varieties), $\tau^{\sigma-1}$ (combination of iceberg trade costs and elasticity of substitution, known as trade freeness, (Baldwin et al. 2003), θ and $\tilde{\theta}$ (Cobb-Douglas shares of labor input in the variable and fixed components of production), κ (degree of fixed costs in terms of the input bundle), and ϖ (Cobb-Douglas share of consumer expenditure).

2.1. Environment

The model incorporates two countries: a capital-abundant "Home" country and a relatively labor-abundant "Foreign" country (denoted with an asterisk $*$). Capital is a relatively more abundant factor in the Home country compared to the Foreign country.

All cross-country differences arise solely from their differing market sizes and factor endowments. All other characteristics, such as technology and preferences, are assumed to be identical across the countries. For analytical simplicity, we assume all firms within a given industry are identical, eliminating the need for a firm-specific index. This allows us to focus on the impact of factor endowments on trade patterns.

Table 1 summarizes the model's parametric assumptions.

Preferences. There are a continuum of industries indexed by $s \in [0, 1]$, with monopolistically competitive firms in each industry. The consumer expenditure shares in industry s are constant by $\varpi(s) \in (0, 1)$, such that $\int_0^1 \varpi(s)\,\mathrm{d}s = 1$. The elasticity of substitution across products within industry s is constant and denoted by $\sigma(s) > 1$.

Table 1: Model Assumptions

Parameter	Assumption	Implication
Market size	$Y > 1$	Home = large country
Factor endowment ratio	$\frac{K/L}{K^*/L^*} > \frac{\max_s\{\zeta(s)\}}{\min_s\{\zeta(s)\}}$	Capital is relatively abundant in Home
Elasticity of substitution	$\sigma(s) > 1$	(i) Markup: $\mu(s) > 1$
across products		(ii) Effective trade cost: $\phi(s) > 1$
Output elasticity	$\alpha(s) < \sigma(s)/[\sigma(s) - 1]$	Unique interior solution for firms
Others	$\kappa(s), \alpha(s) > 0$ and $\tau(s) > 1$	Trivial or by definition
	$\theta(s), \tilde{\theta}(s) \in [0, 1]$	
	$\varpi(s) \in (0, 1)$ and $\int_0^1 \varpi(s)\, \mathrm{d}s = 1$	

Notes: The parameter $\zeta(s)$ is defined in equation (3)

Trade costs. To export one unit of a good, a firm must ship $\tau(s) > 1$ units of the good in industry s. The effective trade cost is denoted by $\phi(s) > 1$, satisfies $\phi(s) = [\tau(s)]^{\sigma(s)-1}$, and represents free trade. Following Hanson and Xiang (2004), $\phi(s)$ is given rather than $\tau(s)$.

Production technology. In each industry, we employ an isoelastic production function that includes both labor and capital inputs, priced at w and r, respectively. Labor and capital input shares in variable costs are $\theta(s)$ and $1 - \theta(s)$, respectively. These can differ from the fixed cost function's input shares, $\tilde{\theta}(s)$ and $1 - \tilde{\theta}(s)$. The total output level, $q(s)$, is the sum of output sold domestically ($q_D(s)$) and exported ($q_X(s)$), i.e., $q(s) = q_D(s) + \tau(s) q_X(s)$. A firm's total cost function and the corresponding marginal cost function (i.e., $\partial \mathrm{tc}(s)/\partial q(s)$) are

$$\mathrm{tc}(s) = \underbrace{\left[w^{\theta(s)} r^{1-\theta(s)}\right] \left[q(s)\right]^{\frac{1}{\alpha(s)}}}_{\text{variable costs}} + \underbrace{\left[w^{\tilde{\theta}(s)} r^{1-\tilde{\theta}(s)}\right] \kappa(s)}_{\text{fixed costs}}, \tag{1}$$

$$\mathrm{mc}(s) = \frac{1}{\alpha(s)} \left[w^{\theta(s)} r^{1-\theta(s)}\right] \left[q(s)\right]^{\frac{1}{\alpha(s)}-1}, \tag{2}$$

where the marginal cost functions in the domestic and export markets are $\partial \mathrm{tc}(s)/\partial q_D(s) = \mathrm{mc}(s)$ and $\partial \mathrm{tc}(s)/\partial q_X(s) = \tau(s) \times \mathrm{mc}(s)$. We assume that $\alpha(s) < \sigma(s)/[\sigma(s) - 1]$ to ensure a unique interior solution for firm optimization. This condition implies that an individual firm's marginal cost curve is steeper than its demand curve, ensuring the finiteness of optimal firm decisions.

The main parameter of interest, $\alpha(s)$ (the output elasticity of variable inputs), is equal to the average variable cost divided by the marginal cost.[2] The output elasticity allows non-constant marginal costs: $\alpha(s) > 1$ or < 1 indicates a decreasing or increasing slope of the marginal cost curve, respectively. Also, $\kappa(s)$ represents fixed costs in terms of the input bundle.[3] Our cost structure with fixed costs and non-constant marginal costs generates variable returns to scale, measured by the average (total) cost divided by the marginal cost.

The cost structure in Krugman (1979, 1980); Hanson and Xiang (2004) is a special case of our cost structure with $\alpha(s) = 1$ and $\tilde{\theta}(s) = \theta(s) = 1$.[4] Also, $\alpha(s) = 1$, $\theta(s) \in (0, 1)$ and $\tilde{\theta}(s) = 1$ yields Grossman and Helpman (1991)'s cost function.

Country sizes and factor endowments. Both factor markets (labor and capital) are perfectly competitive and immobile across countries. We denote Home and Foreign labor input endowments by L and L^*. Similarly, capital endowments in Home and Foreign are K and K^*.

Without loss of generality, we assume that Home is larger than Foreign: Home income (GDP, $Y = wL + rK$) is larger than Foreign income ($Y^* = w^*L^* + r^*K^*$). To simplify our analysis and focus on relative differences in factor prices and endowments, we normalize foreign income (Y^*) to 1. Our assumptions regarding country size and factor endowments imply that Home owns more of at least one factor of production than Foreign.

Additionally, we assume that labor and capital represent the relatively scarce and abundant factors in a large country (Home) compared to a small country (Foreign). Specifically, the Home country's capital-labor ratio, K/L, significantly exceeds the Foreign country's, K^*/L^*, by more than the cross-industry variation in capital-labor intensities ($\zeta(s)$), measured by the two extreme values: the maximum of $\zeta(s)$ divided by the minimum of $\zeta(s)$.

$$\zeta(s) \stackrel{\text{def}}{=} \frac{\{\alpha(s)[1-\theta(s)] + [1-\alpha(s)][1-\tilde{\theta}(s)]\}[\sigma(s)-1] + 1 - \tilde{\theta}(s)}{\{\alpha(s)\theta(s) + [1-\alpha(s)]\tilde{\theta}(s)\}[\sigma(s)-1] + \tilde{\theta}(s)} \tag{3}$$

The intensity ratio, $\zeta(s)$, as defined in the above equation, represents the ratio of equilibrium capital to labor employed in industry s. (A detailed explanation of $\zeta(s)$ is provided in the following

[2] The output elasticity of variable inputs is constant in the isoelastic production function. Let $l_v(s)$ and $k_v(s)$ denote labor and capital inputs in production, and $l_f(s)$ and $k_f(s)$ denote capital and labor inputs in entry/operation. By duality of production and cost functions, the corresponding production functions satisfy $q(s) \propto \{[l_v(s)]^{\theta(s)}[k_v(s)]^{1-\theta(s)}\}^{\alpha(s)}$ and $\kappa(s) \propto [l_f(s)]^{\tilde{\theta}(s)}[k_f(s)]^{1-\tilde{\theta}(s)}$, where $[l_v(s)]^{\theta(s)}[k_v(s)]^{1-\theta(s)}$ and $[l_f(s)]^{\tilde{\theta}(s)}[k_f(s)]^{1-\tilde{\theta}(s)}$ are variable and fixed input bundles.

[3] Because $\kappa(s)$ is exogenously given by definition, adding the scale effect in fixed costs does not affect our model properties. For example, we can define $\tilde{\kappa}(s) = [\kappa(s)]^{\tilde{\alpha}}$ and replace $\kappa(s)$ by $\tilde{\kappa}(s)$. This transformation does not matter for the results.

[4] In Krugman (1979, 1980), the outside sector (perfectly competitive) faces $\alpha(s) = 1$, $\tau(s) = 1$, $\theta(s) = 1$, and $\kappa(s) = 0$.

subsection.) This condition guarantees that the Home country is unequivocally capital abundant relative to Foreign, regardless of industry distribution.

2.2. Firm Optimization and Entry

Under the assumptions in Table 1, each firm's profit maximization problem has a unique interior solution given the aggregate variables. The well-known price-setting condition implies that the domestic and export prices of each product variety in industry s are $p_D(s) = \mu(s)\mathrm{mc}(s)$ and $p_X(s) = \mu(s)[\tau(s)\mathrm{mc}(s)]$, respectively, where $\mu(s) = \sigma(s)/[\sigma(s)-1]$ represents the identical markup (price over marginal costs) in both domestic and export markets. Then, operating profits (revenue minus variable costs) can be expressed as

$$\underbrace{\left[1-\frac{\alpha(s)}{\mu(s)}\right]p(s)q(s)}_{\text{operating profits}} = \underbrace{\left[1-\frac{\alpha(s)}{\mu(s)}\right]p_D(s)q_D(s)}_{\text{domestic operating profits}} + \underbrace{\left[1-\frac{\alpha(s)}{\mu(s)}\right]p_X(s)q_X(s)}_{\text{export operating profits}} \tag{4}$$

$$= \left[\frac{\mu(s)}{\alpha(s)}-1\right]\left[w^{\theta(s)}r^{1-\theta(s)}\right][q(s)]^{\frac{1}{\alpha(s)}}, \tag{5}$$

where $p(s) = p_D(s) = \mu(s)\mathrm{mc}(s)$ denotes the domestic price and the export price before accounting for iceberg trade costs.[5]

Traditional HME studies have assumed that marginal costs are unaffected by production scale (i.e., $\alpha(s) = 1$), leading to linear separability between domestic and export outputs in the cost function (equation 1). Additionally, the marginal cost function (equation 2) remains constant regardless of total, domestic, or export output levels. This characteristic allows firms to independently make production decisions in domestic and export markets without considerations of internal firm interdependence. In contrast, the introduction of non-constant marginal costs establishes within-firm connections between domestic and export markets. For instance, if a firm experiences decreasing marginal costs (i.e., $\alpha(s) > 1$), its domestic market marginal costs decrease with increasing exports. Exporting enables the firm to reduce production costs and prices domestically, leading to heightened demand, sales, and profits. Fixed costs, however, cannot create such interlinkages. Consequently, the interdependence between domestic and export markets resulting from non-constant marginal costs offers a crucial perspective on the role of economics of

[5]Because the variable and marginal cost functions are not linearly separable in the domestic and export market quantities ($q_D(s)$ and $q_X(s)$), it is not straightforward to separate the domestic and export profits. To obtain the first line of equation (6), we assume that the ratio of variable costs in the domestic market to that in the export market equals to the ratio of the domestic market revenue to the export market revenue. See Kim (2021) for more details and discussion.

scale in shaping trade patterns.

In each industry s, the number (mass) of firms in each industry, denoted by $n(s)$, is endogenously determined by the following free entry condition:

$$\left[w^{\tilde{\theta}(s)} r^{1-\tilde{\theta}(s)}\right] \kappa(s) = \left[\frac{\mu(s)}{\alpha(s)} - 1\right] \left[w^{\theta(s)} r^{1-\theta(s)}\right] [q(s)]^{\frac{1}{\alpha(s)}}, \tag{6}$$

such that fixed costs (left-hand side) are equal to operating profits (right-hand-side). Because this condition determines the representative firm's size within an industry, it plays a crucial role in influencing international competitiveness, economics of scale, and HME. We will discuss the relevant mechanism in Section 3.

Firm entry and optimization within each industry equalize the ratio of capital to labor costs across Home and Foreign.

$$\frac{rk(s)}{wl(s)} = \frac{r^* k^*(s)}{w^* l^*(s)} = \zeta(s) \stackrel{\text{def}}{=} \frac{\{\alpha(s)[1-\theta(s)] + [1-\alpha(s)][1-\tilde{\theta}(s)]\}[\sigma(s)-1] + 1 - \tilde{\theta}(s)}{\{\alpha(s)\theta(s) + [1-\alpha(s)]\tilde{\theta}(s)\}[\sigma(s)-1] + \tilde{\theta}(s)}. \tag{7}$$

In the standard homothetic cost structure ($\theta(s) = \tilde{\theta}(s)$), the capital-labor cost ratio simplifies to $1/\theta(s) - 1$, independent of cost- and production-side scale returns ($\sigma(s) - 1$ and $\alpha(s)$).

The common ratio, $\zeta(s)$ defined in equation (3), is determined by the equilibrium factor intensities in production and fixed cost functions. The first part of the numerator and denominator, $\{\alpha(s)[1-\theta(s)] + [1-\alpha(s)][1-\tilde{\theta}(s)]\}[\sigma(s)-1]$ and $\{\alpha(s)[1-\theta(s)] + [1-\alpha(s)][1-\tilde{\theta}(s)]\}[\sigma(s)-1]$, respectively represents the equilibrium intensities of capital and labor in production (variable costs). While the intensity of fixed costs does not influence the factor ratio under constant returns to scale ($\alpha(s) = 1$), it becomes relevant when marginal costs are non-constant ($\alpha(s) \neq 1$). In the latter case, firm size and returns to scale are determined by $\alpha(s)$ and $\tilde{\theta}(s)$, affecting the equilibrium factor intensity in production. The second part of the numerator and denominator are the factor intensities of capital and labor in fixed costs, $1 - \theta(\tilde{s})$ and $\theta(\tilde{s})$, respectively.

In addition to a constant factor ratio of each industry within a country, the individual firm's capital and labor cost ratio (in variable and fixed costs) in Home to Foreign is:

$$\frac{rk(s)}{r^* k^*(s)} = \frac{rk_v + rk_f}{r^* k_v^* + r^* k_f^*} = \left(\frac{w}{w^*}\right)^{\tilde{\theta}(s)} \left(\frac{r}{r^*}\right)^{1-\tilde{\theta}(s)} = \frac{wl_v + wl_f}{w^* l_v^* + w^* l_f^*} = \frac{wl(s)}{w^* l^*(s)}, \tag{8}$$

where the ratio increases with Home factor prices of labor and capital relative to Foreign with weights $\tilde{\theta}(s)$ and $1 - \tilde{\theta}(s)$. From the above equation, we establish a key link between factor prices and factor demands as follows:

$$\left[\frac{n(s)k(s)}{n(s)l(s)}\right]/\left[\frac{n^*(s)k^*(s)}{n^*(s)l^*(s)}\right] = \left(\frac{w}{r}\right)/\left(\frac{w^*}{r^*}\right), \tag{9}$$

which essentially states that the ratio of a firm's demand for capital relative to labor in the home country, compared to the same ratio for a firm in the foreign country, is directly tied to the ratio of wages to rental rates between the two countries. In simpler terms, if labor is relatively expensive in the home country compared to the foreign country, firms in the home country are likely to substitute away from labor and utilize more capital relative to their foreign counterparts. This industry-level relationship between factor prices and factor demands can have broader implications at the aggregate level. For a detailed derivation (equations 7, 8, and 9), see the derivation of Appendix B.

We assumed a larger market in the Home country with a higher capital-to-labor ratio ($K/L > K^*/L^*$). When factor demand increases in this larger market due to its size, the effect will likely be more pronounced for the scarcer factor—labor endowment in this scenario. As a consequence, the home country's inherently high capital-to-labor endowment ratio ($K/L > K^*/L^*$) is likely to translate into a higher equilibrium wage-to-rent ratio compared to the foreign country ($w/r > w^*/r^*$). In other words, the larger market would drive up the price of the scarcer factor (labor) more than the price of the abundant factor (capital), leading to a higher wage-to-rent ratio in the home country. This can be can be formalized in the following lemma.

Lemma 1 *The high capital-to-labor ratio in Home assumption ($(K/L)/(K^*/L^*) > \max_s\{\zeta(s)\}/\min_s\{\zeta(s)\}$) translates into a higher equilibrium ratio of wages (relatively scarce factor prices) to rental rates (relatively abundant factor prices) in Home compared to Foreign ($w/r > w^*/r^*$).*

Proof. See Appendix B. ■

2.3. Equilibrium

Similar to Krugman (1980), the trade equilibrium is summarized by the relative price of inputs (w/w^* and r/r^*), which can be determined from the goods market clearing condition with balanced trade in each economy.

The goods market clearing condition for industry s in the Home country equates the industry's

GDP to the sum of Home and Foreign demand for the industry's output as

$$n(s)p(s)q(s) = \frac{n(s)[p(s)]^{\sigma(s)-1}}{n(s)[p(s)]^{\sigma(s)-1} + n^*(s)[p^*(s)]^{1-\sigma(s)}/\phi(s)}\varpi(s)Y + \frac{n(s)[p(s)]^{\sigma(s)-1}}{n(s)[p(s)]^{\sigma(s)-1} + n^*(s)[p^*(s)]^{1-\sigma(s)}\phi(s)}\varpi(s). \tag{10}$$

The aggregate accounting for Home is given by

$$G(w/w^*, r/r^*) = Y - \int_0^1 n(s)p(s)q(s)\,\mathrm{d}s = 0, \tag{11}$$

where $G(w/w^*, r/r^*)$ is the aggregation of $g(s)$ with weights $\varpi(s)$, i.e., $G(w/w^*, r/r^*) \overset{\text{def}}{=} \int_0^1 \varpi(s)g(s)\,\mathrm{d}s$. The industry's function $g(s)$ is determined by the following expression:

$$g(s) \overset{\text{def}}{=} \frac{Y}{\phi(s)/h(s) - 1} - \frac{1}{\phi(s)h(s) - 1}, \tag{12}$$

where $h(s)$ is a function of relative factor prices between Home and Foreign as follows:

$$h(s) \overset{\text{def}}{=} \left(\frac{w}{w^*}\right)^{-\{\alpha(s)\theta(s)+[1-\alpha(s)]\tilde{\theta}(s)\}[\sigma(s)-1]-\tilde{\theta}(s)} \left(\frac{r}{r^*}\right)^{-\{\alpha(s)[1-\theta(s)]+[1-\alpha(s)][1-\tilde{\theta}(s)]\}[\sigma(s)-1]-[1-\tilde{\theta}(s)]}. \tag{13}$$

The parameteric assumptions of $\mu(s) = \sigma(s)/[\sigma(s) - 1] > \alpha(s)$ and $\sigma(s) > 1$ imply the negative exponential coefficient of Home wage relative to Foreign, i.e., $\{\alpha(s)\theta(s) + [1 - \alpha(s)]\tilde{\theta}(s)\}[\sigma(s) - 1] + \tilde{\theta}(s) = \{\alpha(s)\theta(s) + [\mu(s) - \alpha(s)]\tilde{\theta}(s)\}[\sigma(s) - 1] > 0$. Similarly, the coefficient of Home rental rate relative to is also negative $\{\alpha(s)[1 - \theta(s)] + [1 - \alpha(s)][1 - \tilde{\theta}(s)]\}[\sigma(s) - 1] + [1 - \tilde{\theta}(s)] > 0$ in equation (13). For a detailed derivation, see Appendix B.

Despite the possibility of non-zero net exports in individual industries, the overall trade is balanced at the aggregate level. The function $g(s)$ represents Home's net-export position in industry s. The aggregate trade should be balanced in equilibrium, i.e., $G(w/w^*, r/r^*) = 0$, even though trade balances can differ across industries ($g(s)$). In equilibrium, the Home input price relative to Foreign should be adjusted to achieve zero aggregate net-exports, $G(w/w^*, r/r^*) = 0$.

Equations (11)–(13) show that a large market size exerts pressure on factor market equilibrium prices to rise. Trade costs make Home a more attractive location for firms due to its larger market. This, in turn, leads to higher demand for production inputs in Home. This leads to relatively higher equilibrium factor prices in Home compared to Foreign.

Suppose that equilibrium factor prices in Home do not exceed those in Foreign, i.e., $w/w^* \leq 1$

and $r/r^* \leq 1$. Then, equation (13) yields $h(s) \leq 1$ in all industries. However, this contradicts the equilibrium condition ($G = \int_0^1 g(s)\,\mathrm{d}s = 0$) under the large market size and high trade costs because $h(s) \geq 1$ and $Y, \phi > 1$ imply $g(s) > 0$ for all industries.

3. Production Scale and Input Structure

This section explores the underlying mechanism wherein the impact of economies of scale, stemming from non-constant marginal costs, depends on an industry's input intensities in variable and fixed cost functions.

To investigate how the difference ($\tilde{\theta}(s) - \theta(s)$) in labor input intensity between fixed and variable costs influences the cross-country variation in firm sizes, we establish the relative production input bundle between Home and Foreign in industry s by using the free entry conditions of both countries (equation 6).

$$\ln \frac{[l_v(s)]^{\theta(s)}[k_v(s)]^{1-\theta(s)}}{[l_v^*(s)]^{\theta(s)}[k_v^*(s)]^{1-\theta(s)}} = \frac{1}{\alpha(s)} \ln \frac{q(s)}{q^*(s)} = \left[\tilde{\theta}(s) - \theta(s)\right] \ln \frac{w/r}{w^*/r^*}, \tag{14}$$

where $[l_v(s)]^{\theta(s)}[k_v(s)]^{1-\theta(s)}$ denotes a firm's input bundle in production (also known as the variable input bundle), which satisfies $q(s) \propto \{[l_v(s)]^{\theta(s)}[k_v(s)]^{1-\theta(s)}\}^{\alpha(s)}$. This input bundle represents the firm's production scale and serves as a measure of firm size.

Specifically, if we include only one input at Home, for instance labor, and both variable and fixed costs have identical input shares ($\tilde{\theta}(s) = \theta(s) = 1$), the input prices cancels out in the free entry condition of equation (6), irrespective of the endowment (size) differences across countries. This leads to identical firm sizes across countries in equation (14). As a result, the industry's output level for the representative firm becomes identical across countries. This cross-country identical firm size eliminates the role of internal economics of scale in shaping trade patterns. Hence, careful consideration of inputs used in production and their relative shares in fixed vs variable costs is crucial in trade models with increasing returns in production.

In our current framework, the higher equilibrium price of the scarce factor relative to that of the abundant factor in the large country ($w/r > w^*/r^*$), suggests that the representative firm is bigger in the large country compared to the small country only when the fixed cost function more intensively uses the large country's scarce factor compared to the variable cost function (equation (14)). The following proposition summarizes this equilibrium characteristic.

Proposition 1 *In the equilibrium factor price of Lemma 1, the relative size of the representative firm in Home's industry s is larger, equal, or smaller than the size of the corresponding Foreign firm if and only if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\tilde{\theta}(s) < \theta(s)$, respectively.*

Proof. The result is obtained using equation (14) with $w/r > w^*/r^*$. ■

When $\tilde{\theta}(s) > \theta(s)$, the large country faces cost advantages for individual firms in industries with decreasing marginal costs. Intuitively, when the fixed cost function (related to entry and operation) requires a higher proportion of the relatively scarce and expensive labor input, (i) fewer firms will enter in the large country, and consequently (ii) each individual firm in the large country will have a larger production scale. Thus, the firm in the large country benefits from its large size in international markets when marginal costs decrease with production.

On the other hand, if $\tilde{\theta}(s) < \theta(s)$, firm entry and operation in the large country require less of the more expensive labor input (relative to the labor input used in production), leading to higher firm entry and a lower production scale for each firm. In this case, decreasing marginal costs in an industry weaken the firm's international competitiveness in the large country due to its smaller firm size.[6]

In summary, Proposition 1 points out the pivotal role played by the interaction between non-constant marginal costs and non-homotheticity in inputs. This interaction, influencing firm sizes and cost advantages, shapes the relationship between country size and economies of scale. We explore this relationship more carefully in the next section in the context of HME.

4. Increasing Returns and Home Market Effects

In this section, we explicitly study how industry characteristics shape specialization patterns between Home and Foreign. To achieve this, we employ the ratio of exports to imports. In particular, industry s exhibits HME if this ratio increases with the relative country size. The ratio of exports to imports in Home (large country) is given by

$$\frac{\mathrm{ex}(s)}{\mathrm{im}(s)} = \left\{ \frac{1 + \phi(s)[n(s)/n^*(s)][p(s)/p^*(s)]^{1-\sigma(s)}}{1 + \phi(s)[n^*(s)/n(s)][p(s)/p^*(s)]^{\sigma(s)-1}} \right\} \frac{1}{Y}, \tag{15}$$

[6]Appendix A provides more details on the implications for international competitiveness arising from firm sizes.

which can be expressed by terms of the relative input price and the relative GDP as:

$$\frac{\mathrm{ex}(s)}{\mathrm{im}(s)} = \left\{ \frac{1+\phi(s)H(s)}{1+\phi(s)/H(s)} \right\} \frac{1}{Y} \quad \text{where } H(s) = \frac{\left\{[\phi(s)]^2 Y + 1\right\} h(s) - (Y+1)\phi(s)}{[\phi(s)]^2 + Y - (Y+1)\phi(s)h(s)}. \tag{16}$$

In the above equation, $h(s)$ is defined in equation (13). For a detailed derivation, see Appendix B.

Equations (16) and (13) clearly illustrate that as industry characteristics interact with relative input price terms, an appreciated price of the scarce input (labor) relative to abundant input (capital) at Home leads to different impacts on net-exports and location across industries, determining the direction of HME.[7] In particular, the output elasticity ($\alpha(s)$, and the input intensity difference ($\tilde{\theta}(s) - \theta(s)$) are crucial determinants of industrial HME, as formally stated by the following proposition. Additionally, the subsequent lemma formally asserts that fixed costs do not play a role in equations (16) and (13), indicating their limited impact on HME.

Lemma 2 *In equilibrium, Home exports to imports ratio in industry s is independent of $\kappa(s)$.*
Proof. This follows directly from the fact that $\kappa(s)$ disappears in equations (16) and (13). ■

Proposition 2 *In equilibrium, Home exports to imports ratio in industry s is increasing, constant, or decreasing in $\alpha(s)$ if and only if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\tilde{\theta}(s) < \theta(s)$, respectively.*
Proof. See Appendix B. ■

Scale (dis) economies in production can affect HME through two channels: fixed costs and non-constant marginal costs. The output elasticity of variable inputs α, representing non-constant marginal costs, affects HME. On the other hand, economics of scale from fixed costs have no impact on the direction of industry-level HME in our model, where fixed cost functions are symmetric across countries in each industry.

The degree of increasing returns from non-constant marginal costs positively impacts HME when fixed costs are more Home scarce input intensive than variable costs. Industries with high $\alpha(s)$ tend to specialize in the larger country ($\mathrm{ex}(s)/\mathrm{im}(s) > 1$ at Home) if $\tilde{\theta}(s) > \theta(s)$. Conversely, when $\tilde{\theta}(s) < \theta(s)$, Foreign tends to specialize in such industries. This result also holds for the relative number of firms ($n(s)/n^*(s)$), another measure of HME.

Corollary 1 *In equilibrium, Home to Foreign ratio of the number of firms in industry s is increasing, constant, or decreasing in $\alpha(s)$ if and only if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\tilde{\theta}(s) < \theta(s)$, respectively. However, the ratio is independent of $\kappa(s)$.*

[7] More precisely, equation (16) indicates that the ratio of exports to imports increases with $H(s)$. In equation (16), $H(s)$ increases with $h(s)$, which, in turn, decreases with w/w^* (as shown in equation 13). For $w/w^* > r/r^*$, $h(s)$ is increasing, constant, or decreasing in $\alpha(s)$ if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\tilde{\theta}(s) < \theta(s)$, respectively.

Proof. See Appendix B. ■

Understanding the mechanisms behind the result on scale economies relies on the relative firm size determined by free entry conditions (Proposition 1), that was discussed in Section 3. A higher output elasticity $\alpha(s)$ indicates that internal scale economics in production are more pronounced in an industry. In such cases, a country with a larger firm size enjoys advantages, regardless of its country size. According to Proposition 1, the impact of a large country size (at Home) on firm size hinges on the factor intensity of the industry's fixed cost function compared to the variable cost function. If this fixed cost function is more intensive in the factor that is relatively scarce and thus expensive in large countries, then larger firm sizes will be observed in the Home country. In other words. $\tilde{\theta}(s) > \theta(s)$ is a required condition for firms at Home to gain an advantage due to its larger country size, resulting in industry specialization and positive net-exports in these industries.[8] Therefore, the degree of non-constant marginal costs captured by $\alpha(s)$, and its interaction with the relative intensities of fixed and variable costs ($\tilde{\theta}(s) - \theta(s)$), determine whether Home or Foreign specializes in a particular industry.

Proposition 2 and Corollary 1 also underscore the importance of introducing multi-inputs and factor endowments for understanding the implications of HME. When considering only a single input, implying $\tilde{\theta}(s) = \theta(s) = 1$, economics of scale becomes irrelevant for HME. Therefore, including only labor as a production input while ignoring other inputs like capital and material costs would lead to an incomplete understanding of the role of non-constant marginal costs in influencing trade and location patterns.

5. Concluding Remarks

We build upon the two-country multi-industry new trade model of Hanson and Xiang (2004) by introducing a more general cost structure. This includes non-constant marginal costs and the incorporation of multiple inputs with heterogeneous factor endowments across countries. Our model delivers some novel insights related to the impact of scale economies on HME patterns. Its role depends on (i) the source (i.e., non-constant marginal costs vs. fixed costs) and (ii) the input intensity of the relatively scarce and abundant factors in variable and fixed cost functions. When large country scarce inputs (relatively higher prices than those in small countries) are more intensively used in variable costs than fixed costs, our model predicts that small countries specialize

[8]The opposite holds if the fixed cost function is less labor input-intensive compared to variable costs, i.e., $\tilde{\theta}(s) < \theta(s)$, in which case Foreign tends to specialize in these industries instead.

in industries with decreasing marginal costs, i.e., a negative relationship between industrial net-exports and increasing returns via non-constant marginal costs. Conversely, the relationship is positive when variable costs use large country abundant inputs more intensively than fixed costs. While the magnitude of fixed costs play a role in determining equilibrium conditions, their influence on the HME is limited.

Our model's predictions suggest a potential issue of omitted variable bias when critical aspects of the cost structure, such as non-constant marginal costs and the input intensities of costs, along with their interactions, are ignored. However, empirical testing of these predictions can be challenging due to the limited availability of data on input intensities in variable and fixed costs. In addressing this challenge, Bak, Kim and Mehra (2022) conducted an examination of the impact of various industry features on HME in the US manufacturing market. To account for cost structures, they attempt to estimate input intensities in variable and constant costs using Shephard's lemma. Their findings, after controlling for cost structures, reveal that industries with high markups and/or trade costs tend to concentrate in large countries, consistent with prior literature. Importantly, their empirical results provide validation for the predictions made by our model.

Our model employs a Krugman-type model, which does not consider endogenous export market participation, as examined in Melitz (2003). In cases where only productive firms engage in endogenous exports, even with decreasing marginal costs, some firms export despite facing negative export market profits. Their decision to export, in turn, lowers their marginal costs, ultimately increasing their domestic and overall profits. As a potentially fruitful avenue for future research, extending our analysis to incorporate endogenous export market entry, as in Melitz (2003); Arkolakis, Costinot and Rodríguez-Clare (2012), may unveil a new connection between the HME and non-constant marginal costs via extensive margins. Another potentially promising direction for future research is to consider sequential moves in oligopolistic markets. With decreasing marginal costs, the timing of export decisions can generate interesting strategic behavior in export markets.[9]

[9]In situations where increasing returns result in decreasing marginal costs, initial exporters can gain an advantage through market preoccupation, which allows them to become larger and more productive compared to latecomers. Their competitiveness and profitability decline when early entrants are already established exporters, reducing their willingness to enter the market. These early entrants tend to supply a significant quantity of products, effectively crowding out potential followers and establishing formidable entry barriers.

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Appendix

A. International Price Competitiveness

This section describes how firm size, cost structure, and their interaction shape international competitiveness within each industry. To do that, we leverage the export-import price ratio, which captures the relative marginal cost of production in Home and Foreign, i.e., $p_X(s)/p_X^*(s) = \mathrm{mc}(s)/\mathrm{mc}^*(s)$. The impact of labor input prices on the relative marginal costs is further elucidated by considering the relative firm size as expressed by the following equation:

$$\ln \frac{p_X(s)}{p_X^*(s)} = \ln \frac{\mathrm{mc}(s)}{\mathrm{mc}^*(s)} = \theta(s) \ln \frac{w}{w^*} - [\alpha(s) - 1] \ln \frac{[l_v(s)]^{\theta(s)}[k_v(s)]^{1-\theta(s)}}{[l_v^*(s)]^{\theta(s)}[k_v^*(s)]^{1-\theta(s)}}, \tag{A1}$$

which is derived from the marginal costs of both countries (equation 2) by incorporating the definition of firm size (input bundle in production).

The last term on the right-hand side of equation (A1) shows how firm sizes and economics of scale (stemming from non-constant marginal costs), influence international competitiveness. In cases where Home industry s boasts a larger production scale relative to Foreign, its price competitiveness (the inverse of the export-import price ratio) increases or decreases depending on whether its firm's marginal costs decrease ($\alpha(s) > 1$) or increase ($\alpha(s) < 1$) with its firm size, respectively. This mechanism, combined with Proposition 1, leads to the following corollary.

Corollary 2 *In the equilibrium of Lemma 1, the logarithmic Home export-import price ratio in industry s is decreasing, constant, or increasing in $\alpha(s)$ if and only if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\theta(s) > \tilde{\theta}(s)$, respectively.*

Proof. The result is obtained by inserting equation (14) into equation (A1). ■

This corollary provides insights into how the interaction between input intensity and output elasticity determines industrial international competitiveness across countries with different country sizes. In industries where fixed costs exhibit a higher intensity of labor inputs (relatively scarce at Home) compared to variable costs, industries with high output elasticity (indicative of decreasing marginal costs) experience a relatively milder erosion of price competitiveness when the equilibrium price of inputs appreciates more at Home than Foreign.

This phenomenon arises because the higher labor input intensity of fixed costs leads to larger firm sizes in the Home country, as discussed in Proposition 1 with equation (14). In this scenario, the advantage goes to industries with decreasing marginal costs ($\alpha(s) > 1$) in the Home country due to their larger sizes relative to Foreign.

Consequently, equilibrium price competitiveness at Home relative to Foreign is either reinforced or attenuated in industries with high $\alpha(s)$ when $\tilde{\theta}(s) > \theta(s)$ or $\tilde{\theta}(s) < \theta(s)$, respectively. This mechanism sheds further light on the importance of input intensities in connecting country size and non-constant marginal costs.

B. Derivations and Proofs

All derivations and proofs are inspired by Hanson and Xiang (2004).

Derivation of the Number of Firms. First, using equations (14) and (A1), we write the derive Home export-import price ratio as a function of the relative price of inputs as

$$\frac{p_X(s)}{p_X^*(s)} = \frac{\mathrm{mc}(s)}{\mathrm{mc}^*(s)} = \left(\frac{w}{w^*}\right)^{\alpha(s)\theta(s)+[1-\alpha(s)]\tilde{\theta}(s)} \left(\frac{r}{r^*}\right)^{\alpha(s)[1-\theta(s)]+[1-\alpha(s)][1-\tilde{\theta}(s)]}. \tag{A2}$$

Then, rewrite $n(s)$ as a function of Y, w/w^*, and r/r^*. Equations (6) and (10) imply

$$\begin{aligned}
&\frac{\kappa(s)\tilde{n}(s)}{1-\alpha(s)/\mu(s)} \\
&\quad = n(s)p(s)q(s) \\
&\quad = \frac{\varpi(s)Yn(s)}{n(s)+n^*(s)[p_X(s)/p_X^*(s)]^{\sigma(s)-1}/\phi(s)} + \frac{\varpi(s)n(s)}{n(s)+n^*(s)[p_X(s)/p_X^*(s)]^{\sigma(s)-1}\phi(s)},
\end{aligned} \tag{A3}$$

where $\tilde{n}(s) \stackrel{\text{def}}{=} w^{\tilde{\theta}(s)}r^{1-\tilde{\theta}(s)}n(s)$. Inserting equation (A2) into the above, we obtain

$$\frac{\kappa(s)\tilde{n}(s)}{1-\alpha(s)/\mu(s)} = \frac{\varpi(s)Y\phi(s)\tilde{n}(s)}{\phi(s)\tilde{n}(s)+\tilde{n}^*(s)/h(s)} + \frac{\varpi(s)\tilde{n}(s)}{\tilde{n}(s)+\phi(s)\tilde{n}^*(s)/h(s)} \tag{A4}$$

Also, the world goods market clearing condition with $r = r^*$ is

$$\frac{\kappa(s)\left[\tilde{n}(s)+\tilde{n}^*(s)\right]}{1-\alpha(s)/\mu(s)} = \varpi(s)(Y+1), \tag{A5}$$

which yields the number of firms to be

$$n(s) = \frac{\tilde{n}(s)}{w^{\tilde{\theta}(s)}r^{1-\tilde{\theta}(s)}} = \frac{Y[\phi(s)]^2-(Y+1)\phi(s)/h(s)+1}{[\phi(s)]^2-\phi(s)\left[1/h(s)+h(s)\right]+1}\left[\frac{1-\alpha(s)/\mu(s)}{\kappa(s)w^{\tilde{\theta}(s)}r^{1-\tilde{\theta}(s)}}\right]. \tag{A6}$$

Derivation of equation (11). Insert equation (A6) into $n(s)p(s)q(s)$.

$$0 = Y - \int_0^1 \frac{Y[\phi(s)]^2 - (Y+1)\phi(s)/h(s) + 1}{[\phi(s)]^2 - \phi(s)\,[1/h(s) + h(s)] + 1} \varpi(s)\,\mathrm{d}s \tag{A7}$$

Because $Y = \int_0^1 \varpi(s) Y\,\mathrm{d}s$, we obtain the result.

$$0 = \int_0^1 \frac{Y\,[\phi(s) - 1/h(s)] - [1/h(s)]\,[\phi(s)/h(s) - 1]}{[\phi(s) - 1/h(s)]\,[\phi(s) - h(s)]} \varpi(s)\,\mathrm{d}s \tag{A8}$$

$$= \int_0^1 \left\{ \frac{Y}{\phi(s)/h(s) - 1} - \frac{1}{\phi(s)h(s) - 1} \right\} \varpi(s)\,\mathrm{d}s = \int_0^1 \varpi(s) g(s)\,\mathrm{d}s \tag{A9}$$

Using the definition of $h(s)$ and equation (A2), we obtain the result.

Derivation of Equations (7), (8), and (9). In industry s, fixed costs of capital and labor inputs are written as: $rk_f = [1 - \tilde{\theta}(s)] w^{\tilde{\theta}(s)} r^{1-\tilde{\theta}(s)} \kappa(s)$ and $wl_f = \tilde{\theta}(s) w^{\tilde{\theta}(s)} r^{1-\tilde{\theta}(s)} \kappa(s)$. Also, variable costs of capital and labor inputs are written as: $rk_v = [1 - \theta(s)] w^{\theta(s)} r^{1-\theta(s)} [q(s)]^{1/\alpha(s)}$ and $wl_v = \theta(s) w^{\theta(s)} r^{1-\theta(s)} [q(s)]^{1/\alpha(s)}$. Using equation (6), we can replace $[q(s)]^{1/\alpha(s)}$ by factor prices. Then, $rk_v = [1 - \theta(s)] w^{\tilde{\theta}(s)} r^{1-\tilde{\theta}(s)} \kappa(s)/[\mu(s)/\alpha(s) - 1]$ and $wl_v = \theta(s) w^{\tilde{\theta}(s)} r^{1-\tilde{\theta}(s)} \kappa(s)/[\mu(s)/\alpha(s) - 1]$. Since fixed cost components share the similar structure as variable costs with respect to factor prices, we can obtain the total capital cost (rk) and total labor cost (wl) by combining variable and fixed costs: $rk = rk_v + rk_f = \{[1 - \theta(s)]/[\mu(s)/\alpha(s) - 1] + 1 - \tilde{\theta}(s)\} w^{\theta(s)} r^{1-\theta(s)} \kappa(s)$ and $wl = wl_v + wl_f \{\theta(s)/[\mu(s)/\alpha(s) - 1] + \tilde{\theta}(s)\} w^{\theta(s)} r^{1-\theta(s)} \kappa(s)$. With straightforward algebra, these equations lead to equations (7) and (8). Equation (9) is directly from equation (8).

Proof of Lemma 1. We begin with equation (7). The equation can be re-written as $rk(s) = \zeta(s) wl(s)$. By aggregation $rK = r \int n(s)k(s)\,\mathrm{d}s = w \int \zeta(s) n(s) l(s)\,\mathrm{d}s \leq w \int n(s) l(s)\,\mathrm{d}s \times \max_s\{\zeta(s)\} = wL \max_s\{\zeta(s)\}$. In Foreign, $r^* K^* = r^* \int n^*(s) k^*(s)\,\mathrm{d}s = w^* \int \zeta(s) n^*(s) l^*(s)\,\mathrm{d}s \geq w^* \int n^*(s) l^*(s)\,\mathrm{d}s \times \min_s\{\zeta(s)\} = w^* L^* \min_s\{\zeta(s)\}$. Thus, $(w/r)/(w^*/r^*) \geq [(K/L)/(K^*/L^*)] \times [\min_s\{\zeta(s)\}/\max_s\{\zeta(s)\}]$. Thus, $(K/L)/(K^*/L^*) > \max_s\{\zeta(s)\}/\min_s\{\zeta(s)\}$ implies $(w/r)/(w^*/r^*) > 1$.

Derivation of Equation (16). The ratio of export to import is $\mathrm{ex}(s)/\mathrm{im}(s) = (1/Y)\,[1 + \phi(s)H(s)]\,/\,[1 + \phi(s)/H(s)]$ where $H(s) \stackrel{\text{def}}{=} [\tilde{n}(s)/\tilde{n}^*(s)]\,h(s)$. By using equation (A6), $H(s)$ can be rewritten as a function of $h(s)$ in equation (16).

Proof of Proposition 2. In equation (16), $H(s)$ is increasing in $h(s)$ because $\phi(s) > 1$:

$$\frac{\partial H(s)}{\partial h(s)} = \frac{[\phi(s) - 1]^2 Y}{\{[\phi(s)]^2 + Y - (Y+1)\phi(s)h(s)\}^2} > 0. \tag{A10}$$

For given $w/w^* > r/r^*$, $h(s)$ is increasing, constant, or decreasing in $\alpha(s)$ if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\tilde{\theta}(s) < \theta(s)$, respectively. Hence, $w/w^* > r/r^*$ implies that $H(s)$ and $\mathrm{ex}(s)/\mathrm{im}(s)$ are increasing, constant, or decreasing in $\alpha(s)$ if $\tilde{\theta}(s) > \theta(s)$, $\tilde{\theta}(s) = \theta(s)$, or $\tilde{\theta}(s) < \theta(s)$, respectively.

Proof of Corollary 1. It is sufficient to show that Home to Foreign ratio increases with $h(s)$ and is independent of $\kappa(s)$. The number of firms in equation (A6) implies Home to Foreign ratio to be

$$\frac{n(s)}{n^*(s)} = \left[\frac{Y[\phi(s)]^2 - (Y+1)\phi(s)/h(s) + 1}{[\phi(s)]^2 - (Y+1)\phi(s)h(s) + Y}\right]\left[\left(\frac{w}{w^*}\right)^{-\tilde{\theta}(s)}\left(\frac{r}{r^*}\right)^{\tilde{\theta}(s)-1}\right], \tag{A11}$$

which is increasing in $h(s)$ and does not include $\kappa(s)$.